PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

3

Kya-Kya padhna hai -

3.1 Introduction

3.2 Graphical Method of Solution of a Pair of Linear Equations

3.3 Algebraic Methods of Solving a Pair of Linear Equations

3.3.1 Substitution Method

3.3.2 Elimination Method

3.4 Summary

Pair of linear Equation in Two Variables

A pair of linear equations in two variables is **two equations that use the same two variables**. The general form of a pair of linear equations is ax + by + c = 0, where a, b, and c are real numbers.

Example —

(1) $ax + by + c = 0$

(2) $px + qy + c = 0$

Where - a, b and c is a real no.

Methods (a pair of linear Equation) -

① → Graphically method / Algebraic method

i → Intersecting lines / exactly one solⁿ (unique)

ii → coincident lines / Infinitly many solⁿ

iii → Parallel lines / no solⁿ

② → Algebraic method

i → Substitution method

ii → Elimination method

iii → Cross multiplication method (×)

① → Graphically method

i → intersecting lines / one sol^n or unique

A pair of linear equations in two variables, which has a solution, is called a consistent pair of linear equations.

formula – $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

1.	$x - 2y = 0$ $3x + 4y - 20 = 0$	$\frac{1}{3}$	$\frac{-2}{4}$	$\frac{0}{-20}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution (unique)

$x - 2y = 0$ (a_1, b_1)

$3x + 4y - 20 = 0$ (a_2, b_2, c_2)

→ $\frac{1}{3} = \frac{-2}{4}$ → $\frac{1}{3} \neq -\frac{1}{2}$ → unique sol^n / exactly one sol^n

Example 1 : Check graphically whether the pair of equations

$$x + 3y = 6 \quad (1)$$

and

$$2x - 3y = 12 \quad (2)$$

is consistent. If so, solve them graphically.

Example 2 : Graphically, find whether the following pair of equations has no solution, unique solution or infinitely many solutions:

$$5x - 8y + 1 = 0 \qquad (1)$$

$$3x - \frac{24}{5}y + \frac{3}{5} = 0 \qquad (2)$$

EXERCISE 3.1

1. Form the pair of linear equations in the following problems, and find their solutions graphically.

   (i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost ₹ 50, whereas 7 pencils and 5 pens together cost ₹ 46. Find the cost of one pencil and that of one pen.

2. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(i) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

2. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(ii) $9x+3y+12=0$

$18x+6y+24=0$

2. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(iii) $6x - 3y + 10 = 0$

$2x - y + 9 = 0$

3. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5$; $2x - 3y = 7$ (ii) $2x - 3y = 8$; $4x - 6y = 9$

3. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$; $9x - 10y = 14$

(iv) $5x - 3y = 11$; $-10x + 6y = -22$

3. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(v) $\frac{4}{3}x + 2y = 8$; $2x + 3y = 12$

4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(i) $x + y = 5, \quad 2x + 2y = 10$

(ii) $x - y = 8, \quad 3x - 3y = 16$

(iii) $2x + y - 6 = 0, \quad 4x - 2y - 4 = 0$

(iv) $2x - 2y - 2 = 0, \quad 4x - 4y - 5 = 0$

5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

6. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(i) intersecting lines

(ii) parallel lines

(iii) coincident lines

7. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

ii → coincident lines / infinitely many solⁿ

A pair of linear equations which are equivalent has infinitely many distinct common solutions. Such a pair is called a dependent pair of linear equations

formula — $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

2.	$2x + 3y - 9 = 0$ $4x + 6y - 18 = 0$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{-9}{-18}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions

$2x + 3y - 9 = 0$ (a_1, b_1, c_1)

$4x + 6y - 18 = 0$ (a_2, b_2, c_2)

→ $\frac{2}{4} = \frac{3}{6} = \frac{-9}{-18}$

$\frac{1}{2} = \frac{1}{2} = \frac{1}{2}$ → coincident lines, infinitely many solⁿ iii → Parallel lines / no soln

A pair of linear equations which has no solution, is called an inconsistent pair of linear equations.

formula – $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

3.	$x + 2y - 4 = 0$ $2x + 4y - 12 = 0$	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{-4}{-12}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

Soln →

$x + 2y - 4 = 0$ (a_1, b_1, c_1)

$2x + 4y - 12 = 0$ (a_2, b_2, c_2)

→ $\frac{1}{2} = \frac{2}{4}$ but $\frac{+4}{+12} = \frac{1}{3}$

$\frac{1}{2} = \frac{1}{2} \neq \frac{1}{3}$

↳ No soln (Inconsistent)

Sari baaten Ek Jagah —

(i) intersecting, then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. → Unique / Exactly one solⁿ

(ii) coincident, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. → Infinitly many solⁿ

(iii) parallel, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. → no solⁿ

③ → Algebraic method

i → Substitution method

substituted the value of one variable by expressing it in terms of the other variable to solve the pair of linear equations. That is why the method is known as the substitution method.

ii → Elimination method

The method used in solving the example above is called the elimination method, because we eliminate one variable first, to get a linear equation in one variable.

iii → Cross multiplication method (×)

Example 4 : Solve the following pair of equations by substitution method:

$$7x - 15y = 2 \quad (1)$$

$$x + 2y = 3 \quad (2)$$

Example 5 : Solve the following question—Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically by the method of substitution.

Example 6 : In a shop the cost of 2 pencils and 3 erasers is ₹9 and the cost of 4 pencils and 6 erasers is ₹18. Find the cost of each pencil and each eraser.

Example 7 : Two rails are represented by the equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$. Will the rails cross each other?

1. Solve the following pair of linear equations by the substitution method.

(i) $x + y = 14$

$x - y = 4$

(ii) $s - t = 3$

$\frac{s}{3} + \frac{t}{2} = 6$

(iii) $3x - y = 3$
$9x - 3y = 9$

(iv) $0.2x + 0.3y = 1.3$
$0.4x + 0.5y = 2.3$

(v) $\sqrt{2}x+\sqrt{3}y=0$

$\sqrt{3}x-\sqrt{8}y=0$

(vi) $\frac{3x}{2}-\frac{5y}{3}=-2$

$\frac{x}{3}+\frac{y}{2}=\frac{13}{6}$

2. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, she buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹ 105 and for a journey of 15 km, the charge paid is ₹ 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

25 km?

(v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Example 8 : The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ₹ 2000 per month, find their monthly incomes.

Example 9 : Use elimination method to find all possible solutions of the following pair of linear equations :

$$2x + 3y = 8 \qquad (1)$$

$$4x + 6y = 7 \qquad (2)$$

Example 10 : The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?

1. Solve the following pair of linear equations by the elimination method and the substitution method :

(i) $x + y = 5$ and $2x - 3y = 4$

(ii) $3x + 4y = 10$ and $2x - 2y = 2$

(iii) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

(iv) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to a bank to withdraw ₹ 2000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 she received.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

3.4 Summary

In this chapter, you have studied the following points:

1. A pair of linear equations in two variables can be represented, and solved, by the:
   (i) graphical method
   (ii) algebraic method

2. Graphical Method :

The graph of a pair of linear equations in two variables is represented by two lines.

(i) If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is **consistent**.

(ii) If the lines coincide, then there are infinitely many solutions — each point on the line being a solution. In this case, the pair of equations is **dependent (consistent)**.

(iii) If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is **inconsistent**.

3. Algebraic Methods : We have discussed the following methods for finding the solution(s) of a pair of linear equations :

(i) Substitution Method

(ii) Elimination Method

4. If a pair of linear equations is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then the following situations can arise :

(i) $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_1}$: In this case, the pair of linear equations is consistent.

(ii) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$: In this case, the pair of linear equations is inconsistent.

(iii) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$: In this case, the pair of linear equations is dependent and consistent.

5. There are several situations which can be mathematically represented by two equations that are not linear to start with. But we alter them so that they are reduced to a pair of linear equations.